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Abstract. Many superconductors show a low temperature electrical resistivity of Fermi liquid type $\rho=AT^2$. We show empirically that there exists a relationship between A and T_c when both vary under an external parameter, such as pressure. The more resistive the compound the higher the T_c. Through the analysis of Landau theory of FL, we find that it is a general feature of FL, due to the fact that the scattering that is the main cause of τ is the same one that bounds the pairs that condensed at T_c. We devise a method that allows the determination of the coupling constant λ, which is validated through application to ^3He-'s superfluid transitions and τ's extracted from different properties. This method works for conventional superconductors, but fails with heavy fermions.

1. Introduction

The standard electrical resistance of a metal is controlled by electron-phonon scattering yielding a $\rho = \alpha T$ dependence at high temperatures and a $\rho = \beta T^n$, $n \sim 5$ dependence at low temperatures. However, in some special materials, the low temperature electrical resistivity is of the type $\rho = AT^2$. It has long been attributed to electron-electron scattering (e-e-s) and is considered as the signature of a Fermi liquid (FL) behavior for the electronic spectra of the studied compound. The term A has been related to the electronic linear term of the specific heat γ of each material through the Rice-Kadowaki-Woods[1,2] relation $A \propto \gamma^2$ through their common dependence on the effective mass m^*. The precise origin of the AT^2 term has been elucidated for certain materials: direct Coulomb scattering for ultra-pure alkali metals[3] or spin fluctuations for heavy fermion materials [4].

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A number of superconductors of different type present the $\rho = AT^2$ dependence above the superconducting transition at T_c: heavy fermions, $A-15$ compounds, borocarbides, etc. It is natural to expect that the pre-transitional scattering is related the pairing interaction, as has been shown, e.g. for aluminum, where the $\rho = AT^2$ has been traced down to phonon mediated e-e-s [5]. In particular, it would be interesting to study the T_c *vs* A correlation. However, this can only be done properly if both properties vary simultaneously under a external parameter, e.g. pressure. A long term study of superconductors, together with the reanalysis of published data has allowed us to determine a direct relationship between the coefficient A and the superconducting transition temperature, T_c, that we present on Fig. 1. It is clear from the figure that T_c is a monotonous increasing function of A, thus the stronger the scattering the stronger the T_c. The empirical relationship that follows from the results shown on this figure has never been reported (nor even addressed).

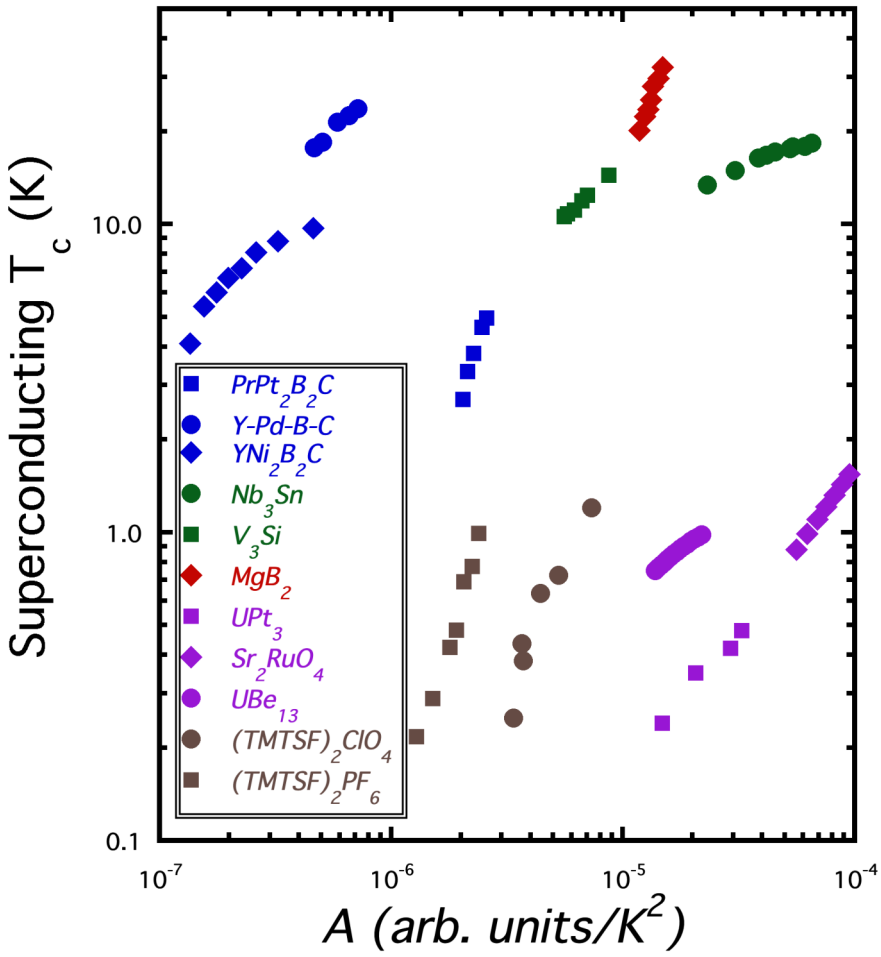

Figure 1 Empirical relation between T_c and the coefficient A of the quadratic temperature term for different values of an external parameter is applied, pressure unless specified otherwise. $PrPt_2B_2C$(blue squares)[6]; *Y-Pd-B-C*(blue dots)[7]; YNi_2B_2C(blue diamonds)[Our data]; Nb_3Sn (α or e^- irradiation, green dots)[8]; V_3Si (green squares)[Our data]; MgB_2(red diamonds)[9]; UPt_3(magenta squares)[10]; Sr_2RuO_4(magenta diamonds)[11]; UBe_{13}(magenta dots)[12]; $(TMTSF)_2ClO_4$ (brown dots)[13],$(TMTSF)_2PF_6$ (brown squares)[13].

2. Theory

We can understand this relationship within Landau theory for Fermi liquids as follows.

2.1. Theoretical relationship between τ and T_c

We reduce the inverse quasiparticle-quasiparticle scattering time and the transition temperature to the superfluid phase, to the Landau amplitude scatterings within the *s-p* approximation. From standard Landau FL theory[14] we obtain the inverse quasiparticle-quasiparticle scattering time τ^{-1},

$$\tau^{-1} = \frac{m^{*3} < W(\theta,\phi) > (k_B T)^2}{8\pi^4 \hbar^6} = \frac{m^* < W(\theta,\phi) > N\left(k_F\right)^2 (k_B T)^2}{8\pi^2 \hbar^2 V^2 E_F} \tag{1}$$

Where m^* is the effective mass, V the volume, $N(k_F)$ the density of states at the Fermi level, $< W(\theta,\phi) >$ is the transition probability which describes the scattering of two quasiparticles whose momenta are related by the standard angles θ and ϕ, the bracket indicating the angle average through the Abrikosov-Khalatnikov angles; k_B the Boltzmann constant and $\hbar$ the Planck's constant. Developing in Legendre polynomials within the *s-p* approximation[15] we obtain for the triplet transition probability

$$W_{\uparrow\uparrow}(\theta,\phi) = \frac{2\pi}{\hbar}\left|\frac{A_{\uparrow\uparrow}(\theta,\phi)}{N(k_F)}\right|^2 \sim \left|\frac{(1+A_0{}^a)}{N(k_F)}\right|^2 \tag{2}$$

and

$$(\tau \cdot T^2)^{-1} \sim \frac{m^* k_B{}^2 \left(1+A_0^a\right)^2}{4\pi\hbar^3 V^2 E_F} \propto \left(1+A_0^a\right)^2 \tag{3}$$

where $A_{\uparrow\uparrow}(\theta,\phi)$ is the triplet scattering amplitude and T_F the Fermi temperature and A_l^σ the Landau scattering amplitudes (we have taken A_0^s=1). A similar result, $(\tau \cdot T^2)^{-1} \propto \left(2A_1^{ep}\right)^2$, is obtained for singlet scattering taking $A_l^s = A_l^a = A_l^{ep}$, as in this case the interaction does not depend on spin direction.

Now, following Ref. [16], Patton and Zaringhalam [17] (PZ) estimated the transition temperature to a condensed state as a function of the Landau scattering amplitudes, T_F being the Fermi temperature and $\lambda_{\uparrow\uparrow}$ the triplet coupling constant, and α a parameter,

$$T_c = 1.13\alpha T_F \mathrm{e}^{1/\lambda_{\uparrow\uparrow}}$$

$$\lambda_{\uparrow\uparrow} = A_{\uparrow\uparrow}(\pi,\phi)/3\cos(\phi) = \sum_l (-)^l (A_l{}^s + A_l{}^a)/12 \approx -(1+A_0{}^a)/6 \tag{4}$$

where we have limited the development to $l = 0,1$ within the *s-p* approximation and we have neglected singlet scattering. For singlet scattering, neglecting triplet scattering, we have $\lambda_{\uparrow\downarrow} \approx 2A_1^{ep}$. From (3) and (4) we conclude that $(\tau T^2)^{-1} \propto \left(\lambda_{\uparrow\uparrow}\right)^2$, the same for singlet scattering. Thus

$$T_c \propto \mathrm{e}^{-\varsigma/\sqrt{(\tau T^2)^{-1}}} \tag{5}$$

2.2. Electrical resistivity formula

The FL resistivity of metals[18] is given by $\rho_{ee} = \frac{m^*}{ne^2\tau_R}$, where n is the carrier density, m^* the effective mass, *e* the electronic charge and $\tau_R = number \times \tau$. From (1) , (2) and (4) we find now that

$$\rho_{ee} \approx \frac{m^{*2} k_B{}^2 \left(1+A_0^a\right)^2}{ne^2 4\pi\hbar^3 V^2 E_F} T^2 \qquad A \approx \frac{m^{*2} k_B{}^2}{ne^2 4\pi\hbar^3 V^2 E_F} \lambda_{\uparrow\uparrow}^2 \tag{6}$$

A similar result can be obtained for singlet scattering.

3. The case of Helium 3

We can test formula (5) adapting it for the most studied Fermi liquid, i.e. ^3He; it becomes $T_c / T_F^{**} = \alpha e^{-\varsigma/\sqrt{(\tau T^2)^{-1}}}$ (7), where T_F^{**} is the Fermi temperature renormalized by all the interactions at each pressure. We plot $T_c / T_F^{**}(P)$ data as a function of the thermal conductivity scattering rates [19] and fit with expression (7) with α and ζ as adjustable parameters (Figure 2). We can thus extract the coupling constant $\lambda = \sqrt{(\tau T^2)} / \zeta$. The obtained values $0.2 < \lambda < 0.4$ are in very good agreement with theoretical calculations [20]. We must note, though, that all pressure effects other than those on τ are probably taken into account by the renormalization with $T_F^{**}(P)$.The application on ^3He confirms our analysis based on the idea that the pre-transitional scattering determines T_c.

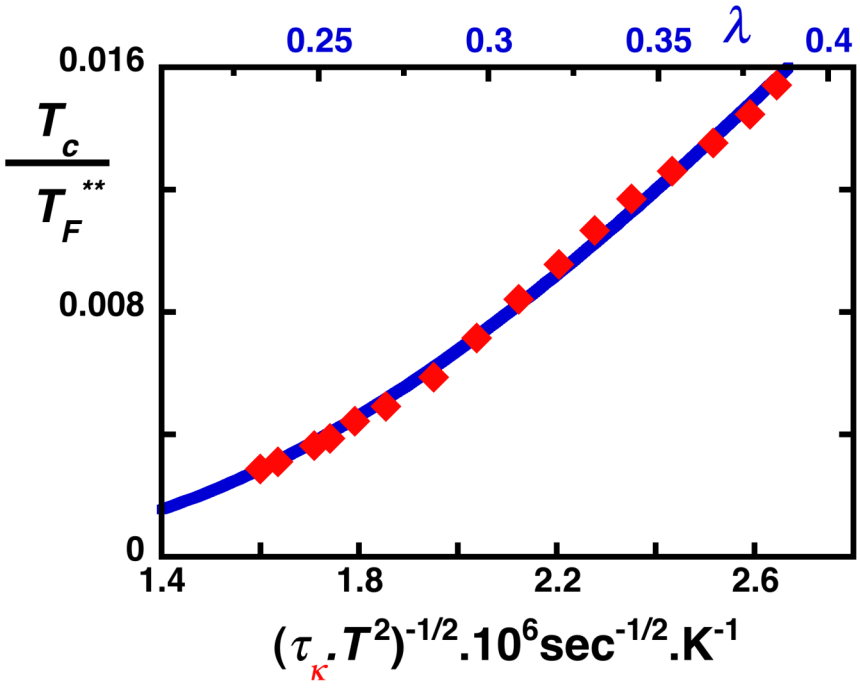

Figure 2 Superfluid transition temperature T_c of ^3He normalized by $T_F^{**}(P)$, the Fermi temperature renormalized by all the interactions at each pressure, as a function of the inverse square root of the scattering time extracted from the thermal conductivity multiplied by the square of the temperature. Fitting with $T_c / T_F^{**} = \alpha . e^{-\varsigma/\sqrt{(\tau T^2)^{-1}}}$ (blue line) allows the determination of the parameter ς and the coupling constant $\lambda = \sqrt{(\tau T^2)^{-1}} / \varsigma$ in excellent agreement with theory [20].

4. Application to superconductors

Having shown the utility of expression (5), we come back to the superconductors of Figure 1. As $A \propto (\tau T^2)^{-1}$, we can now attempt an equivalent type of fit $T_c = \theta e^{-\varsigma/\sqrt{A}}$. We must bear in mind, though, that application of elementary FL to materials with complex Fermi surfaces is bound to be cumbersome. In this case other parameters that are now present besides λ in A, may vary, as well.

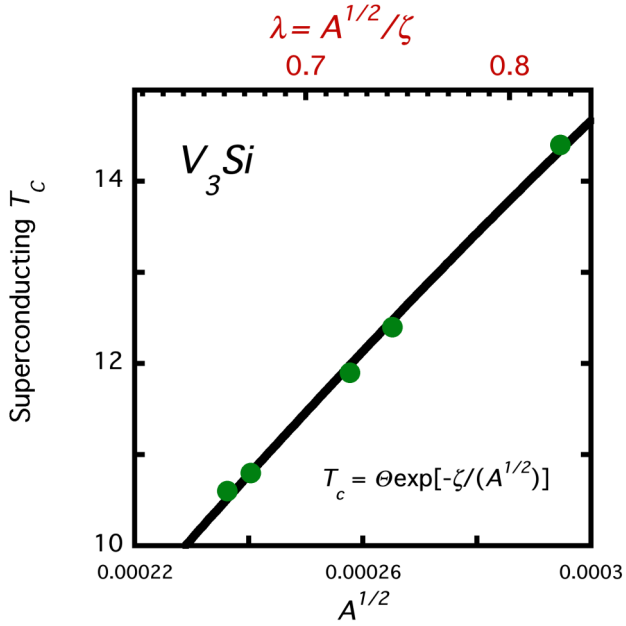

Figure 3 The dependence of T_c on $\sqrt{A}$ and the fit with expression $T_c = \theta e^{\varsigma/\sqrt{A}}$ (solid black) for a *V_3Si* sample (Our data). The agreement of the obtained λ with published data [21] is excellent.

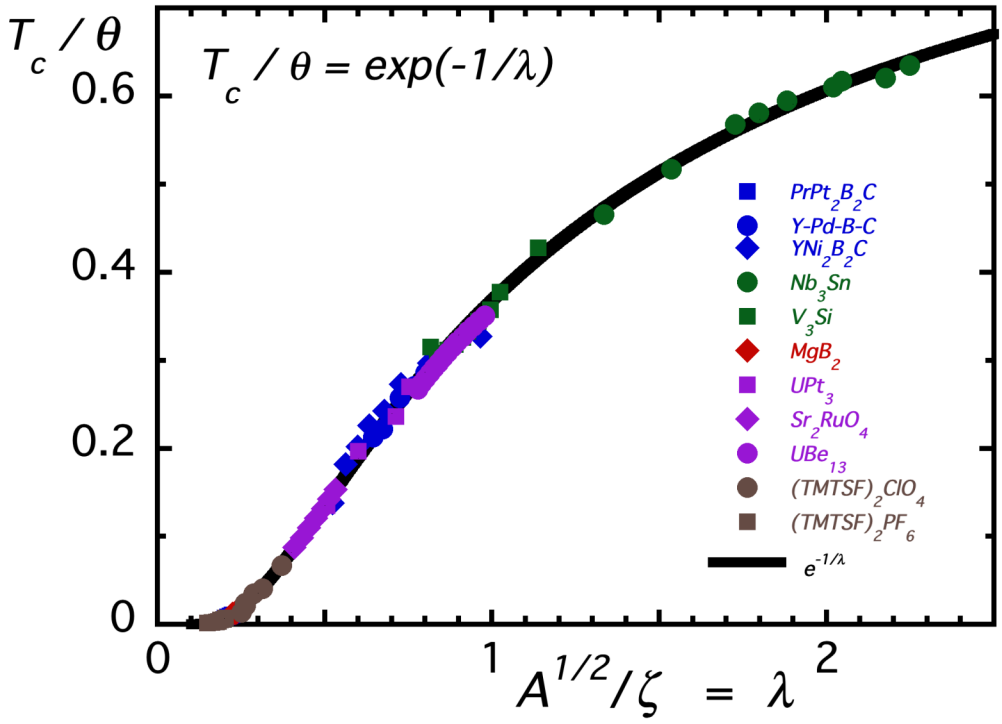

Figure 4 The same type of fit shown for the compounds of Fig. 1, presented now with each T_c normalized to the fitting parameter θ and as a function of $\lambda = \sqrt{A}/\varsigma$. The values of λ we obtain for *A-15* compounds and borocarbides agree within 10% with those obtained from other methods, while those of heavy fermions are much lower than reported previously.

Also, we ignore the pressure dependence of the θ parameter ($\theta \propto 1.13\hbar\omega_D$ for conventional superconductors, where ω_D is the Debye frequency). We show on Fig. 3 an example of a fit on the V_3Si data, that allows us to obtain θ and ζ, and hence the coupling constant $\lambda = \sqrt{A}/\varsigma$. It yields $\lambda \approx 0.87$ at ambient conditions, in excellent agreement with the value obtained from tunneling methods $\lambda = 0.89$ [21]. Thus, the variation of T_c and A seems to be controlled mainly by the variation of τ.

Applying the same procedure to the compounds of Fig. 1, we can obtain for each material the corresponding θ and ζ. Plotting now T_c/θ against $\lambda = \sqrt{A}/\varsigma$ we obtain Fig. 4. All the dispersed data of Fig. 1 collapse onto one single curve, that scales the compounds according to their respective superconducting coupling constants, derived from their resistivity coefficients *A*. For heavy fermions, the variation of *A* with pressure seems to be controlled by m^* not τ, invalidating our method. The fact that our analysis works properly on irradiated Nb_3Sn, suggests that it may be interesting to study the variation of T_c and *A* in heavy fermions by using irradiation, not pressure, as defects should have a stronger effect on the scattering rate τ than on m^*.

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